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Analysis of Sampling Clock Phase Noise in Homodyne FMCW Radar Systems

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Abstract—In many contemporary electronic systems, phase noise sets the bound on the achievable performance. Radar systems are no exception, with the actual radar signals carrying significant amounts of phase noise due to the high transmit frequencies. In coherent radars, some of the phase noise sidebands on the received signal are cancelled due to mixing in the receiver. The sampling clock used to sample the intermediate frequency (IF) signals also introduces phase noise/jitter. This paper focuses on the contribution of the sampling clock's phase noise to the overall phase noise in the sampled signal in coherent homodyne FMCW radar systems. We develop a model relating the phase noise in the sampled signal to the phase noise in the radar signals and the jitter in the sampling clock. We apply our analysis to example FMCW radar systems. The derived model can be used to work out the phase noise requirement on the sampling clock for a given phase noise level in radar signals.

I. INTRODUCTION

Phase noise in the frequency domain, written as $\mathcal{L}(f)$, is defined as one half of the spectral density of phase fluctuations $S_\theta(f)$ having units of rad^2/Hz [1]. The conventional definition of phase noise around a carrier signal is the ratio of the power in the noise sidebands per Hz relative to the power in the carrier, and is specified in dBc/Hz on a plot of the power spectrum. The latter definition is only valid for signals having small phase noise and negligible AM noise [2].

Phase noise appears as phase-modulation sidebands around a carrier's spectrum. For radar systems having a high dynamic range this causes the clutter-floor to increase around large targets making the detection and tracking of small targets impossible in the region of raised clutter-floor [3]. Decreasing the overall phase-noise, therefore, is a prime challenge in high-performance radars. In FMCW radars the phase noise appears as noise-sidebands in range around each target [4]. An additional effect in coherent radars is the cancellation of phase noise at shorter ranges due to coherence.

The effect of phase jitter in sampling clocks has been addressed before as contributing to the overall system noise floor [5], [6], [7], and as the clock's noise spectrum being transferred to a noise-less signal under the sampling process [6], [8]. However, the case of sampling a signal corrupted with phase noise using a clock having its own phase jitter, and their relative contribution to the total phase noise in the sampled signal has been mentioned rarely. In [9] the total phase noise in the sampled signal is accurately estimated using an iterative optimization-based approach. However this approach does not

give insight into the phase jitter requirements of the ADC clock or how the clock jitter compares with the received signal's phase noise. In [10] the problem of the transfer of the sampling clock's noise to a generic input signal has been addressed. However, the relative contributions of the input's phase noise and the clock's phase noise has not been addressed. Ultra-low phase noise oscillators and sampling clocks are expensive, so an estimation of the phase noise requirement is imperative to select the oscillator meeting the phase noise requirement with the lowest cost.

In this paper we present our analysis of the effect of the phase jitter in the analog-to-digital converter's (ADC) sampling clock on the sampled radar signals having their own phase noise. We build on our previous work [11] to model the total phase noise in the demodulated radar signal and take into account the effect of coherent phase noise cancellation in the radar receiver. Afterwards we develop a model for the total phase noise in the sampled radar signal as a function of the phase noise in the demodulated radar signal and the phase noise in the sampling clock. We present a generalized analysis first and then apply the analysis to two FMCW radar systems.

II. SYSTEM DESCRIPTION

Fig. 1 shows a block diagram of the 77 GHz radar system being studied. The *Frequency Synthesizer* block generates a signal synthesized using a phase-frequency detector (PFD)-based phase lock loop (PLL). The synthesized signal is frequency-multiplied to the transmit frequency by the *Transceiver* block. The backscatter from the target is received by the receive-antenna and passed on to the transceiver which demodulates the signal to an intermediate-frequency (IF). The IF signal is digitized after filtering and amplification. Fig. 1 shows the phase noise at various points in the system using the symbol $\mathcal{L}_{sub}(f)$, where *sub* is the subscript showing the phase noise measurement point in the system.

The phase noise on the sampling clock $\mathcal{L}_{CLK}(f)$ and the IF signal $\mathcal{L}_{IF}(f)$ are shown. We will now derive a relationship of these to the total phase noise in the sampled signal.

III. NOISE ANALYSIS

As discussed earlier, the IF signal corrupted with phase noise is sampled using a clock signal having its own phase noise. Let $x(t)$ be the IF signal and $y(mT)$ be the sampled signal, m being the sample number and T being the inverse

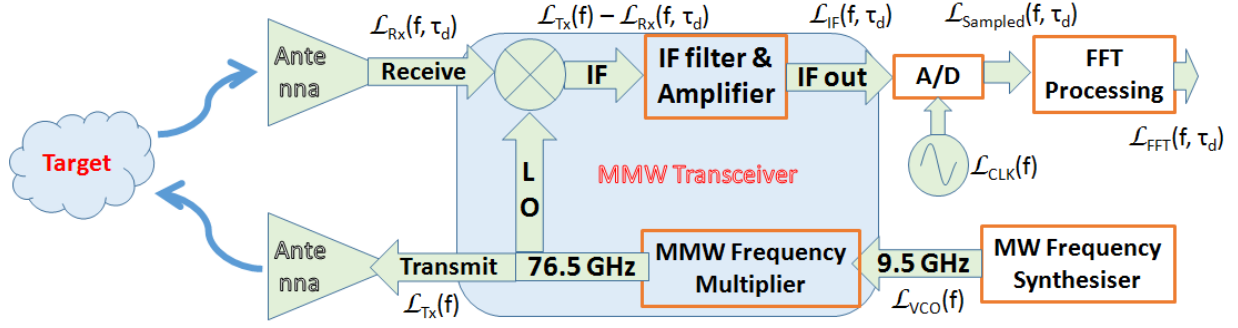


Fig. 1. Block diagram showing phase noise propagation in a FMCW radar system.

of the sampling rate. Using a Taylor Series approximation, it was shown in [10] that the autocorrelation function of $y(mT)$ can be written as,

$$r_y(mT) = r_x(mT) - r_x''(mT) \cdot r_{tj}(mT), \quad (1)$$

where r_{tj} is the autocorrelation of the *time jitter* process on the ADC sampling clock and r_x is the autocorrelation of $x(t)$. The essential conditions for (1) to hold are that $x(t)$ be smooth enough for the existence of a local derivative and the RMS time jitter in the sampling clock, σ_{tj} , be much less than the reciprocal of the maximum signal frequency in $x(t)$, or,

$$\sigma_{tj} \ll \frac{1}{F_{Sigmax}}. \quad (2)$$

We propose that under the same condition, (1) can be extended to the case where $x(t)$ is corrupted by phase noise. This is especially true for sinewaves. Phase jitter essentially causes randomness in the zero-crossings of the waveform [12]. So on the time-scale of phase jitter, the signal's level and its derivatives do not change significantly for a sufficiently smooth function. The same argument holds for a sum of sinewaves, as in the IF signal of a real FMCW radar - if (2) could be satisfied, (1) would still hold.

With this in mind, (1) can be used for a signal $x(t)$ having phase noise that is sampled with a sampling clock having its own phase noise/jitter. The total signal plus noise power in the sampled signal is given by,

$$r_y(0) = r_x(0) - r_x''(0) \cdot r_{tj}(0). \quad (3)$$

Note that $r_{tj}(0)$ can either be measured using a suitable instrument, or representative values can be read directly off oscillator datasheets where the RMS jitter $\sigma_{tj} = \sqrt{r_{tj}(0)}$ is specified. So in our analysis we don't need to use the sampling clock's frequency spectrum to work out the total RMS jitter. In the following we will derive expressions for $r_x(0)$ and $r_x''(0)$.

A. Phase noise in the IF signal

The Frequency Synthesizer block in Fig. 1 generates a 9.5 GHz signal using a PFD-based PLL. Fig. 2 shows a generic phase noise plot of this type of frequency synthesizers. It can be seen that the noise below the loop bandwidth B_L is dominated by the PFD (and not the reference oscillator [8],

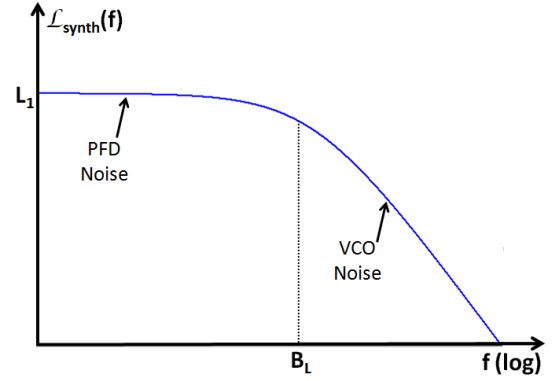


Fig. 2. Phase Noise at the output of a generic PLL synthesizer.

[11]) at a level L_1 dBc/Hz, whereas outside B_L it is dominated by the VCO. Assuming a 20 dB/decade roll-off on the VCO phase noise, a simplified expression for this phase noise plot is [8],

$$\mathcal{L}_{Synth}(f) = \frac{10^{L_1/10}}{1 + \left(\frac{f}{B_L}\right)^2} \quad (4)$$

Due to frequency multiplication by N , the transmitter phase noise can be written as,

$$\mathcal{L}_{Tx}(f) = N^2 \times \mathcal{L}_{Synth}(f). \quad (5)$$

The signal scattered by the target at range R is received at the radar after a delay $\tau_d = 2R/c$, c being the speed of light. The phase noise at the output of the homodyne mixer is given by [13],

$$\mathcal{L}_{IF}(f) = \mathcal{L}_{Tx}(f) \times 4\sin^2(\pi f \tau_d). \quad (6)$$

For small τ_d some of the phase noise is cancelled due to coherence. Using (4) and (5) we can write,

$$\mathcal{L}_{IF}(f) = \frac{4N^2 10^{L_1/10} \times \sin^2(\pi f \tau_d)}{1 + \left(\frac{f}{B_L}\right)^2}. \quad (7)$$

B. Signal model for the noisy IF signal

The IF radar signal $x(t)$ is a sinewave having the phase noise in (7). We can write this signal as,

$$x(t) = A_0 \sin(\omega_0 t + \theta(t)), \quad (8)$$

where $\theta(t)$ is the zero-mean phase noise process. Assuming $\theta(t) \ll 1$, (8) can be written as,

$$x(t) \approx A_0 \sin(\omega_0 t) + A_0 \theta(t) \cos(\omega_0 t). \quad (9)$$

The autocorrelation function of $x(t)$ is,

$$r_x(\tau) = E[x(t)x(t+\tau)]. \quad (10)$$

Inserting (9) we get,

$$r_x(\tau) = E[(A_0 \sin(\omega_0 t) + A_0 \theta(t) \cos(\omega_0 t)) \times (A_0 \sin(\omega_0(t+\tau)) + A_0 \theta(t+\tau) \cos(\omega_0(t+\tau)))]. \quad (11)$$

The expected value of the cross terms are zero, as can be verified. Expanding and computing the expectation we get,

$$r_x(\tau) = \frac{A_0^2}{2} \cos(\omega_0 \tau) (1 + R_\theta(\tau)), \quad (12)$$

where $R_\theta(\tau) = E[\theta(t)\theta(t+\tau)]$ is the autocorrelation of the phase noise process $\theta(t)$. It follows that,

$$r_x(0) = \frac{A_0^2}{2} (1 + R_\theta(0)). \quad (13)$$

The phase noise in the IF signal, $\mathcal{L}_{IF}(f)$, is given by (7). So the spectral density of the $\theta(t)$ is $S_{\theta_{IF}}(f) = 2\mathcal{L}_{IF}(f)$. Computing the inverse Fourier Transform of $S_{\theta_{IF}}(f)$ we get,

$$R_\theta(\tau) = K \left[e^{-2\pi B_L |\tau|} - \frac{1}{2} \left(e^{-2\pi B_L |\tau - \tau_d|} + e^{-2\pi B_L |\tau + \tau_d|} \right) \right] \quad (14)$$

where $K = 4\pi N^2 10^{L_1/10} B_L$. Therefore,

$$R_\theta(0) = K [1 - e^{-2\pi B_L \tau_d}]. \quad (15)$$

Using (12) one may verify that,

$$r_x''(0) = \frac{A_0^2}{2} [-\omega_0^2 - \omega_0^2 R_\theta(0) + R_\theta''(0)]. \quad (16)$$

That is, to compute (16) we need to compute $R_\theta''(0)$. From (14) one can verify that,

$$R_\theta''(0) = (2\pi B_L)^2 R_\theta(0). \quad (17)$$

Therefore,

$$r_x''(0) = \frac{A_0^2}{2} [-\omega_0^2 - \omega_0^2 R_\theta(0) + (2\pi B_L)^2 R_\theta(0)]. \quad (18)$$

The second term in (18) is negligible compared with the first assuming $R_\theta(0) \ll 1$. So we can write,

$$r_x''(0) \approx \frac{A_0^2}{2} [-\omega_0^2 + (2\pi B_L)^2 R_\theta(0)]. \quad (19)$$

Finally, we note that the IF signal's frequency $\omega_0 = 2\pi f_0$ is related to the propagation delay time τ_d as,

$$f_0 = \tau_d \frac{B_S}{T_S}, \quad (20)$$

where B_S and T_S are the swept bandwidth and the sweep time respectively in an FMCW radar. For B_S in the range of 100's of MHz and T_S in the range of milliseconds, f_0 can range from fractions of a kHz to 10's of MHz.

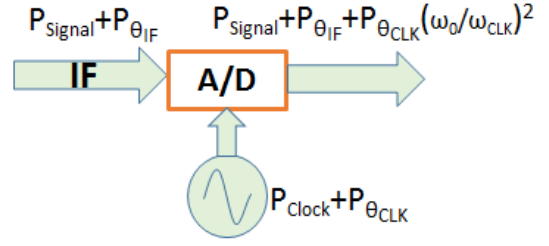


Fig. 3. Illustration of how the clock jitter adds to the IF signal's phase noise.

C. Total noise in the sampled signal

Inserting (13) and (19) in (3) we get,

$$r_y(0) = \frac{A_0^2}{2} (1 + R_\theta(0)) - \frac{A_0^2}{2} (-\omega_0^2 + (2\pi B_L)^2 R_\theta(0)) r_{tj}(0) \quad (21)$$

$$\Rightarrow r_y(0) \approx \frac{A_0^2}{2} + \frac{A_0^2}{2} R_\theta(0) + \frac{A_0^2}{2} \omega_0^2 r_{tj}(0).$$

The first term in (21) is the signal power. The second term is the noise power due to phase noise in the IF radar signal, which we term $P_{\theta_{IF}}$. The third term is the noise power due to the sampling clock, and conforms to a well-known result [7], [8], [14]. As can be noticed, the fourth term has been ignored because σ_{tj} for clocks is specified in pico- or femto-seconds. Computing $r_{tj}(0) = \sigma_{tj}^2$ will make this term minuscule compared with the second term in (21). Equation (21) is an important and powerful result appealing to intuition - the total phase noise is the sum of the phase noise in the IF signal and the phase jitter in the sampling clock scaled by ω_0^2 . We can conclude from (21) that in order to see the effect of sampling clock jitter on the total phase noise, we need to compare the two noise terms. This is summarized in Fig. 3.

IV. APPLICATION TO FMCW RADAR SYSTEMS

We will now analyse the total phase noise in the sampled signal in two example FMCW radar systems working at 77 GHz and 5 GHz respectively. Due to the difficulty in synthesizing a low-noise source at 77 GHz the noise in the IF signal is much higher than in the 5 GHz Microwave (MW) radar. The goal here is to ascertain which of the noise terms in (21) dominates the overall noise in the sampled signal. The noise terms vary with τ_d , i.e., the target range, so it is appropriate to compute them as a function of τ_d (and parametrized by $R_{tj}(0)$).

The system parameters of the two radar systems are shown in Table I. Using those parameters we can compute the noise terms for the two radars as follows:

The 77 GHz Radar:

$$R_\theta(0) = 0.0804 \left(1 - e^{-2\pi 10^5 \tau_d} \right), \quad (22)$$

$$\omega_0^2 r_{tj}(0) = 1.42 \times 10^{25} \tau_d^2 \times r_{tj}(0). \quad (23)$$

TABLE I
PARAMETERS OF THE FMCW RADARS BEING STUDIED

Parameters	MMW Radar	MW Radar
Swept Bandwidth, B_S	600 MHz	500 MHz
Sweep Time, T_S	1 ms	0.5 ms
Loop Bandwidth, B_L	100 kHz	50 kHz
Freq. Multiplication Factor, N	8	1
In-band noise level, L_1	-90 dBc/Hz	-120 dBc/Hz

TABLE II
VARIATION OF THE NOISE TERMS VERSUS TARGET RANGE

	Range (m)	15	150	300	750	1500
	τ_d (μs)	0.1	1	2	5	10
77 GHz Radar	$R_\theta(0)$	0.005	0.0375	0.0575	0.077	0.08
	$\omega_0^2 r_{tj1}(0)$	1.42×10^{-13}	0.142×10^{-10}	$+0.568 \times 10^{-10}$	3.55×10^{-10}	14.2×10^{-10}
	$\omega_0^2 r_{tj2}(0)$	1.42×10^{-11}	0.142×10^{-8}	0.568×10^{-8}	3.55×10^{-8}	14.2×10^{-8}
	$\omega_0^2 r_{tj3}(0)$	1.42×10^{-9}	0.142×10^{-6}	0.568×10^{-6}	3.55×10^{-6}	14.2×10^{-6}
5 GHz Radar	$R_\theta(0)$	3.83×10^{-8}	2.93×10^{-7}	4.5×10^{-7}	6.01×10^{-7}	6.27×10^{-7}
	$\omega_0^2 r_{tj1}(0)$	3.95×10^{-13}	3.95×10^{-11}	1.58×10^{-10}	9.87×10^{-10}	3.95×10^{-9}
	$\omega_0^2 r_{tj2}(0)$	3.95×10^{-11}	3.95×10^{-9}	1.58×10^{-8}	9.87×10^{-8}	3.95×10^{-7}
	$\omega_0^2 r_{tj3}(0)$	3.95×10^{-9}	3.95×10^{-7}	1.58×10^{-6}	9.87×10^{-6}	3.95×10^{-5}

The 5 GHz Radar:

$$R_\theta(0) = 6.28 \times 10^{-7} \left(1 - e^{-2\pi 10^4 \tau_d}\right), \quad (24)$$

$$\omega_0^2 r_{tj}(0) = 3.95 \times 10^{25} \tau_d^2 \times r_{tj}(0). \quad (25)$$

Note that (22) and (24) imply that $R_\theta(0) \ll 1$ for all τ_d as assumed in the previous section to ignore the second term in (18). Table II summarizes the noise terms versus target range. We have considered three sampling clocks as follows:

- 1) $\sigma_{tj1} = \sqrt{r_{tj1}(0)} = 1$ ps
- 2) $\sigma_{tj2} = \sqrt{r_{tj2}(0)} = 10$ ps
- 3) $\sigma_{tj3} = \sqrt{r_{tj3}(0)} = 100$ ps.

In the case of the 77 GHz radar it can be seen that all sampling clocks have a negligible noise contribution compared with the IF signal's inherent phase noise. This result makes the selection of the sampling clock much easy (and cheap). For the 5 GHz radar, however, it can be seen that Clock 1 has lower noise contribution than the IF signal, Clock 2 is comparable, and Clock 3 has a higher noise contribution than the IF signal. It should be noted that for a given radar the noise terms depend directly on the noise parameters in Table I, and not directly on the actual operating frequency of a radar.

From (21) we conclude that, as a figure-of-merit, one noise term dominates the other if it is at least 10 times larger. So the sampling clock's noise contribution must be 10 times less than the radar signal's phase noise to have a minimal effect.

V. CONCLUSION

In this work we analysed the effect of jitter in the sampling clock on radar signals having their own phase noise. We derived an intuitive and powerful equation for the total phase noise in the sampled radar signal. In summary, to select a sampling clock for a given radar system we need to compare the intrinsic phase noise in the IF radar signal with the phase noise transferred from the sampling clock to the IF signal. A

detailed analysis of a higher-noise MMW radar and a lower-noise Microwave radar showed that a lower-cost sampling clock may be adequate for a MMW radar having a noisy IF signal, while a more expensive clock will be needed for a radar with a relatively low-noise IF signal. The analysis can be extended easily to radars operating in other frequency bands.

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